Detection of chatter vibration in a drilling process using multivariate control charts

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Abstract

Time series analysis and multivariate control charts are used to devise a real-time monitoring strategy in a drilling process. The process is used to produce holes with high length-to-diameter ratio, good surface finish and straightness. It is subject to dynamic disturbances that are classified as either chatter vibration or spiralling. A new nonparametric control chart for multivariate processes is proposed. It is used to detect chatter vibration which is dominated by single frequencies. The results showed that the proposed monitoring strategy can detect chatter vibration and that some alarm signals are related to changing physical conditions of the process.

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1. Introduction

Deep hole drilling methods are used for producing holes with a high length-to-diameter ratio, good surface finish and straightness. For drilling holes with a diameter of 20 mm and above, the BTA (boring and trepanning association) deep hole machining principle is usually employed. Fig. 1 shows the working principles of the process. The process is subject to dynamic disturbances that are classified as either chatter vibration or spiralling. Chatter vibration leads to excessive wear of the cutting edges of the tool which has an undesirable effect on the tool life. In extreme cases, it damages the boring wall by causing marks, called chatter marks, on the cylindrical surface of the bore hole, see Fig. 2. Concerning spiralling, it damages the workpiece severely. The defect of shape and surface quality constitutes a significant impairment of the workpiece.

As the deep hole drilling process is often used during the last production phases of expensive workpieces, it is necessary that a process monitoring system be devised to detect these disturbances during the process operation. The purpose of this work is to develop such a real time monitoring strategy by using statistical process control techniques. This strategy is used to detect the transition from stable operation to chatter vibration. In Section 2, models that describe the time varying dynamics of the process are reviewed. In Sections 3 and 4, a monitoring strategy is proposed and applied to real data, respectively.

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Starting bush OilGuide pad
Oil supply device
Stuffing box
Boring bar

Fig. 1. BTA deep hole drilling, working principle.

Boring head Work piece
Oil supply device

Fig. 2. Radial chatter marks.

2. Time varying process dynamics

Several drilling experiments are conducted according to a given experimental design in order to study the dynamics of the process. The feed \( f \) (mm), cutting speed \( v_c \) (m min\(^{-1}\)) and oil flow rate \( \dot{V}_{\text{oil}} \) (l min\(^{-1}\)) are considered as influencing factors. The damper was not considered in order to allow the system to evolve to chatter and spiralling. Note that there are some uncontrollable influences such as the temperature of the boring bar and more importantly the wear of the cutting edges and the guiding pads. During these experiments several on-line measurements were sampled. Chatter is easily recognized in the on-line measurements by a fast increase of the dynamic part of the torque, force and acceleration signals. However, it is the drilling torque measurements that yield the earliest and most reliable information about the transition from stable operation to chatter vibration. For a complete discussion, see Theis (2004).

The spectrograms of the drilling torque, in different experiments, showed clearly that single frequencies dominate the process when chatter vibration is observed. Theis (2004) determined all the relevant frequencies of the process and showed that the most prominent frequency bands are near 234, 703 and 1183 Hz. These frequencies are the eigenfrequencies of the boring bar. For example, Fig. 3 shows the spectrogram of an exemplary process showing varying dynamics states. The process starts with a stable phase during which the BTA tool is guided within the starting bush (depth \( \leq 32 \) mm). After that the process is in different states of chatter that can be distinguished by the three dominating frequencies of approximately 234, 703 and 1183 Hz. Fig. 3 shows transitions from one chatter state to another which is observed in most of the experiments.

2.1. A drilling depth dependency of chatter frequency: a mechanical interpretation

Weinert et al. (2004) noted that a drilling depth dependency of the chatter frequency is observed in all experiments. Chatter around 234 Hz is observed at different stages of the process and only one transition from 234 Hz chatter to
703 Hz chatter occurs. Chatter around frequency 703 Hz is developed only at later stages of the process and there is no transitions from 703 Hz chatter to any other chatter state. Chatter around 1183 Hz is only observed as initial process chatter vibrations and always a transition to 234 or 703 Hz chatter occurs.

The authors showed that the drilling depth dependency of the chatter vibration is caused by slowly varying boundary conditions. They identified the stiffness and damping effects through the contact between the stuffing box within the oil supply device and the boring bar, as the main influencing factors, see Fig. 1. For the drilling process, the stuffing box is used to prevent the lubrication oil to go outside. However, it exerts a constant friction on the boring bar and therefore a damping effect on the torsional modes, 234, 703 and 1183 Hz. The authors showed that in the beginning of the drilling process the damping of the third mode (1183 Hz) due to the stuffing box is close to its minimum. Therefore, the amplitudes of frequency 1183 Hz increases. This explains that in all experiments chatter vibration with 1183 Hz occurred as initial phase chatter. Towards the end of the process, the damping effect of the stuffing box on the second mode (703 Hz) reaches a minimum. This explains that in all experiments chatter vibration associated with mode 703 Hz was only observed towards the end of the process.

2.2. Process models

Weinert et al. (2004) proposed a model based on the van der Pol equation to describe the transition from stable drilling to chatter in one frequency. In fact, they noted that the different kinds of solutions of Eq. (1) qualitatively coincide with the experimentally observed states in the drilling process. They proposed to model the transition by a Hopf bifurcation in the van der Pol equation. Therefore, the drilling torque is described by

$$\frac{d^2 M(t)}{dr^2} + h(t)(b^2 - M(t)^2) \frac{dM(t)}{dr} + w^2 M(t) = W(t),$$

where $M(t)$ is the drilling torque, $b$ and $w$ are constants, $h(t)$ is a bifurcation parameter and $W(t)$ is a white noise process. In this case, Hopf bifurcation occurs in the system when a stable fixed point becomes unstable to form a limit cycle, as $h(t)$ varies from positive to negative values.

Theis (2004) described the main features of the variation of the amplitudes of the relevant frequencies using a logistic function. He showed that his approximation is directly connected to the van der Pol equation proposed by Weinert et al. (2004). In fact, he considered $M(t)$ as a harmonic process

$$M(t) = R(t) \cos(w + \phi),$$
where $\phi$ is the corresponding phase and showed that
\[
2 \frac{dR(t)}{dt} + h(t)R(t) \left( b^2 - \frac{R(t)^2}{2} \right) = \frac{W(t)}{w},
\]
is the amplitude-equation for the differential equation in (1) if there is only one frequency present in the process. Assuming $\Delta t$ is sufficiently small and by discretization of Eq. (2), the observed variation in amplitude of the relevant frequencies may be described by
\[
R_t = \phi_{1,t} R_{t-1} + \phi_{2,t} R_{t-1}^3 + \epsilon_t,
\]
where $\phi_{1,t} = (1 - b^2 h(t)/2)$ and $\phi_{2,t} = h(t)/4$ are time varying parameters and $\epsilon_t$ is normally distributed with mean 0 and variance $\sigma^2$.

2.3. Modelling the amplitudes of relevant frequencies: autoregressive AR(1) approximation

In this section, time series modelling of the amplitudes of the relevant frequencies is used to set up the monitoring strategy. Messaoud (2006) showed that the amplitudes of the relevant frequencies are moderately positively autocorrelated. This autocorrelation is a normal and unremovable part of the process. Then, monitoring $R_t$ may lead to an ineffective strategy that produces out-of-control signals because of the presence of autocorrelation. In this case, residual control charts are adequate SPC procedures suggested by several authors. For example, see Alwan and Roberts (1988) and Montgomery and Mastrangelo (1991). This procedure requires a model of the autocorrelative structure of the data which can be achieved by fitting an appropriate time series model to the observations. The idea behind residual control charts is if the time series model fits the data well, the residuals will be approximately independent. Then, traditional control charts designed to monitor independent observations can be applied to the residuals.

For the monitoring procedure, the model given by Eq. (3) is approximated by its linear autoregressive part AR(1)
\[
R_t = \phi_{1,t} R_{t-1} + \epsilon_t,
\]
and this AR(1) model is used to calculate the residuals. In fact, it is known that the nonlinear term $\phi_{2,t} R_{t-1}^3$ in Eq. (3) only becomes important when there is chatter. The empirical evidence of this approximation is studied in Section 4.1 using real data. As noted, parameters of Eq. (3) are not constant. For this reason, a moving window of length $m$ is used to estimate the AR(1) parameters. Moving window techniques are useful to estimate model parameters which are time varying assuming stationarity only locally. The window moves in each period covering $m$ observations $R_{t-m+1}, R_{t-m+2}, \ldots, R_t$. In each window, parameters $\phi_1$, $\beta$ and $\sigma^2$ of the linear regression model
\[
R_t = \beta_t + \phi_{1,t} R_{t-1} + \epsilon_t.
\]
are estimated. Note that $\beta$ is included because there is a general shift in the amplitudes after depth 32 mm due to a change in the physical conditions of the process, see Section 4.3. The residuals are calculated using the estimates of the regression parameters $\phi_1$ and $\beta$ at time $t-k$, $k \geq 1$. That is,
\[
\epsilon_t = R_t - \hat{\phi}_{1,t-k} R_{t-1} - \hat{\beta}_{t-k},
\]
The use of $\hat{\phi}_{1,t-k}$ and $\hat{\beta}_{t-k}$ in Eq. (5) is motivated by the fact that using the estimated parameters at time $t$ to calculate the residuals may rather serve to mask changes than to detect them, see Section 5.1. In the following, the residuals are calculated using $k = 5$.

3. A new multivariate control chart based on data depth

Liu (1995) was the first who used the concept of data depth to construct a nonparametric control chart for monitoring processes with multivariate quality measurements. In this section, we propose a new nonparametric exponentially weighted moving average (EWMA) control chart based on sequential ranks of data depth measures for multivariate processes. The proposed chart is a generalization of the nonparametric EWMA for individual observations proposed by Hackl and Ledolter (1992). This chart is used to jointly monitor several relevant frequencies.
3.1. Data depth

Data depth measures how deep (or central) a given point \( X \in \mathbb{R}^d \) is with respect to (w.r.t.) a probability distribution \( F \) or w.r.t. a given data cloud \( S = \{Y_1, \ldots, Y_m\} \). There are several measurements for the depth of the observations, such as Mahalanobis depth, the simplicial depth, half-space depth, and the majority depth of Singh; see Liu et al. (1999). In this work, the Mahalanobis depth and simplicial depths are considered.

(1) The Mahalanobis depth (MD) of a given point \( X \in \mathbb{R}^d \) w.r.t. \( F \) is defined to be

\[
MD(F, X) = \frac{1}{1 + (X - \mu_F)^T \Sigma_F^{-1} (X - \mu_F)},
\]

where \( \mu_F \) and \( \Sigma_F \) are the mean vector and dispersion matrix of \( F \), respectively. The sample version of MD is obtained by replacing \( \mu_F \) and \( \Sigma_F \) with their sample estimates. In fact, how deep \( X \) is w.r.t. \( F \) is measured by how small its quadratic distance is to the mean.

(2) The simplicial depth was introduced by Liu (1990) and revised by Burr et al. (2004).

**Definition 1 (Revised simplicial depth for the sample version (Burr et al., 2004)).** Given a data set \( S = \{Y_1, \ldots, Y_m\} \), the simplicial depth of a point \( X \) is the average of the fraction of closed simplices containing \( X \) and the fraction of open simplices containing \( X \), that is

\[
SD(S, X) = \frac{1}{2} \left( \frac{m}{d + 1} \right)^{-1} \sum_{1 \leq i_1 < \cdots < i_d+1 \leq m} I(X \in s[Y_{i_1}, \ldots, Y_{i_d+1}]) + I(X \in \text{int}(s[Y_{i_1}, \ldots, Y_{i_d+1}])),
\]

where \( I(\cdot) \) is the indicator function, \( s[Y_1, \ldots, Y_{d+1}] \) is a \( d \)-dimensional closed simplex whose vertices are observations \( \{Y_1, \ldots, Y_{d+1}\} \) from \( S \) and \( \text{int} \) refers to the open relative interior of \( s[Y_1, \ldots, Y_{d+1}] \). Equivalently, this could be formulated as, \( SD(S, X) = \rho(S, X) + (1/2)\sigma(S, X) \), where \( \rho(S, X) \) is the number of simplices with data points as vertices which contain \( X \) in their open interior, and \( \sigma(S, X) \) is the number of simplices with data points as vertices which contain \( X \) in their boundary.

3.2. The proposed rank based multivariate EWMA (rMEWMA) control chart

The rMEWMA control chart is used to monitor the residuals vectors \( e_t = (e_{t,1}, \ldots, e_{t,d}) \) over time. Let \( RS = \{e_{t-m+1}, \ldots, e_t\} \) denote a reference sample comprised of the \( m > 1 \) most recent residual vectors. It is used to decide whether or not the process is still in-control at time \( t \). The depths \( D(RS, e_t), i = t-m+1, \ldots, t \), are calculated w.r.t. \( RS \). The sequential rank \( Q_t^s \) is the rank of \( D(RS, e_t) \) among \( D(RS, e_{t-m+1}), \ldots, D(RS, e_t) \). That is,

\[
Q_t^s = 1 + \sum_{i=t-m+1}^t I(D(RS, e_t) > D(RS, e_i)),
\]

where \( I(\cdot) \) is the indicator function. For tied observations, we used the midrank, see Gibbons and Chakraborti (1992). In fact, the simplicial depth is a discrete measure and ties may occur. Especially, there always exist at least \( (d+1)/2m \) extreme points that share the minimum simplicial depth of \( (d+1)/2m \). The standardized sequential rank \( Q_t^m \) is given by

\[
Q_t^m = \frac{2}{m} \left( Q_t^s - \frac{m+1}{2} \right).
\]

The control statistic \( T_t \) is the EWMA of standardized ranks, computed as follows

\[
T_t = \min\{B, (1 - \lambda)T_{t-1} + \lambda Q_t^m\},
\]

\( t = 1, 2, \ldots \), where \( B > 0 \) is a reflection boundary, \( T_0 \) is a starting value, usually set equal to zero, and \( 0 < \lambda \leq 1 \) is a smoothing parameter. The process is considered in-control as long as \( T_t \geq h \), where \( h < 0 \) is a lower control limit.
In fact, we consider a lower one sided rMEWMA control chart because $Q_m$ is “higher the better”. Indeed, a high value of $Q_m$ means that observation $X_t$ is deep w.r.t. RS which refers to a process improvement.

A reflecting boundary is included to prevent the rMEWMA control chart from drifting to one side indefinitely. It is known that EWMA schemes can suffer from an “inertia problem” when there is a process change some time after monitoring begins. That is, an EWMA control statistic may wander away from a center line in a direction opposite to that of a shift that occurs some time after the start of monitoring. In this unhappy circumstance, an EWMA scheme can take long time to signal.

4. Application

In this section, the rMEWMA control charts are used to jointly monitor amplitudes of frequencies 703 and 1183 Hz in a real drilling process application. Note that only two frequencies are considered. This is explained by the computational aspect of the simplicial depth, see Section 5.3. The data are obtained in an experiment with feed $f = 0.185$ mm, cutting speed $v_c = 90$ m min$^{-1}$ and amount of oil $V_{oil} = 300$ l min$^{-1}$. The drilling torque are recorded with a sampling rate of 20 000 Hz. They are divided into segments of length 4096 observations to calculate the periodograms. Thus, an amplitude of each frequency is obtained at each 0.3 mm of drilling. For more details about these choices, see Theis (2004). The data consist of 1000 observations (depth $\leq 300$ mm). In fact, the effect of the chatter vibration is apparent on the bore hole wall after depth 300 mm. Therefore, the transition from stable drilling to chatter vibration is expected to start before that depth and it is not sudden but increasing with time; see Weinert et al. (2004). Note that in this experiment chatter vibration is dominated by frequency 703 Hz.

4.1. Diagnostic checks of the residuals

In the previous section it is indicated that the variation in amplitude of the relevant frequencies, given by Eq. (3), is approximated by an AR(1) model within each time window. Recall that the approximation is valid since the nonlinear term $\phi_2, R^3_{t-1}$ in Eq. (3) is not important before chatter. This assumption is checked using the (Teräsvirta et al., 1993) statistical test for nonlinear dependence. This test is used for nonlinear residual structure, after the linear structure has been removed by fitting the AR(1) model. The idea behind this test is that by fitting the linear AR(1) model to the data, the inherent nonlinear structure will remain in the residuals. Time windows of length 100 observations are used to test for neglected nonlinearity in Eq. (4). Table 1 shows the calculated test statistic and $p$-values for the residuals from fitting the model given by Eq. (4) to the amplitudes of frequencies 703 and 1183 Hz. The first 100 residuals are

<table>
<thead>
<tr>
<th>Hole (mm)</th>
<th>Observation number</th>
<th>Frequency 703 Hz</th>
<th></th>
<th>Frequency 1183 Hz</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TLG test</td>
<td>Ljung–Box test</td>
<td>TLG test</td>
<td>Ljung–Box test</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Test statistic</td>
<td>$p$-Value</td>
<td>Test statistic</td>
<td>$p$-Value</td>
</tr>
<tr>
<td>0–30</td>
<td>0–100</td>
<td>0.74</td>
<td>0.69</td>
<td>0.10</td>
<td>0.75</td>
</tr>
<tr>
<td>30–60</td>
<td>101–200</td>
<td>3.48</td>
<td>0.17</td>
<td>6.96</td>
<td>0.01**</td>
</tr>
<tr>
<td>60–90</td>
<td>201–300</td>
<td>1.60</td>
<td>0.45</td>
<td>0.06</td>
<td>0.81</td>
</tr>
<tr>
<td>90–120</td>
<td>301–400</td>
<td>3.75</td>
<td>0.15</td>
<td>0.17</td>
<td>0.68</td>
</tr>
<tr>
<td>120–150</td>
<td>401–500</td>
<td>2.16</td>
<td>0.34</td>
<td>0.26</td>
<td>0.61</td>
</tr>
<tr>
<td>150–180</td>
<td>501–600</td>
<td>0.58</td>
<td>0.75</td>
<td>0.84</td>
<td>0.36</td>
</tr>
<tr>
<td>180–210</td>
<td>601–700</td>
<td>0.19</td>
<td>0.91</td>
<td>0.89</td>
<td>0.34</td>
</tr>
<tr>
<td>210–240</td>
<td>701–800</td>
<td>7.69</td>
<td>0.02**</td>
<td>1.46</td>
<td>0.23</td>
</tr>
<tr>
<td>240–270</td>
<td>801–900</td>
<td>4.02</td>
<td>0.13</td>
<td>1.07</td>
<td>0.30</td>
</tr>
<tr>
<td>270–300</td>
<td>901–1000</td>
<td>1.03</td>
<td>0.60</td>
<td>2.14</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Note: ** and * denote significant values at the 5% and 10% confidence level, respectively.
calculated using equation
\[ e_t = R_t - \hat{\phi}_{1,100} R_{t-1} - \hat{\beta}_{100}, \]
where \( \hat{\phi}_{1,100} \) and \( \hat{\beta}_{100} \) are estimates of the regression parameters \( \phi_1 \) and \( \beta \) at time 100. As mentioned in section 2.3, \( k = 5 \) is used in Eq. (5).

Table 1 shows that the null hypothesis of linearity of the amplitudes of the two frequencies is rejected only at depth segments where it is known that there is a change in the physical conditions of the process, see Section 4.3. This confirms that the nonlinear term is not important during stable operation of the process.

Moreover, the independence assumption is checked. In fact, if the AR(1) model fits the data well, the residuals will be “approximately” independent. As mentioned, this is a basic assumption for the application of the rMEWMA control charts considered in this work. In fact, it is known that the performance of control charts is affected by the autocorrelation in the observations. In our process, the presence of autocorrelation in the residuals can lead to excessive false alarms and to unnecessary process adjustments. This is destructive to the success and reliability of the proposed monitoring strategy. The Ljung–Box test is used to test the independence assumption of the residuals. The same time windows of length 100 observations used for the Teräsvirta-Lin-Granger nonlinearity test are considered. The results are reported in Table 1. Similar to nonlinearity test, the independence assumption of the residuals is rejected only at depth segments where it is known that there is a change in the physical conditions of the process.

In conclusion, during stable drilling the amplitude of the relevant frequencies of the process can be approximated by the AR(1) model given by Eq. (4) and the residuals are independent.

4.2. Choice of the control charts parameters

The statistical design of the rMEWMA control chart refers to choices of combinations of \( \lambda \), \( h \) and \( B \). It ensures the chart performance meets certain statistical criteria. These criteria are often based on aspects of the run length distribution of the control chart. The run length is defined as the number of observations that are needed to exceed the control limit for the first time. The most common measure of control chart performance is the expected value of the run length; i.e. the average run length (ARL). The ARL should be large when the process is statistically in-control (in-control ARL) and small when a shift has occurred (out-of-control ARL).

The parameters of the rMEWMA control charts are chosen so that they all have an in-control ARL equal to 370. This choice should avoid excessive false alarm signals since the control charts are applied to 1000 observations. Typical values of \( \lambda \) are in the range of \( 0.1 \leq \lambda \leq 0.3 \); see Hackl and Ledolter (1992). In this work, we used \( \lambda = 0.1, 0.2 \) and \( 0.3 \). The corresponding values for \( h \) are, respectively, \(-0.314, -0.475 \) and \(-0.591 \). For the reflecting boundary \( B = -h \) is used. An integral equation is used to approximate the in-control ARL, see Messaoud (2006). The revised simplicial depth is computed using a revised algorithm of Rousseeuw and Ruts (1996), see Section 5.3.

4.3. Results

Table 2 shows the out-of-control signals of the different control charts used to monitor the residuals. All control charts signal at \( 32 \leq \text{depth} \leq 50 \) mm. It is known that the guiding pads of the BTA tool leave the starting bush approximately at depth 32 mm. From previous experiments, the process has been observed to either stay stable or start with chatter vibration, see Weinert et al. (2004). This change in the physical condition of the process induces a sudden increase in the amplitudes of the relevant frequencies. In fact, before that depth, the vibrations of the tool are strongly damped by the starting bush supporting the guiding pads and the amplitudes are low. The fact that the guiding pads leave the starting bush induces a sudden change in the dynamics of the process, caused by the tool being freed. This explains that all control charts have picked up these changes very quickly. Note that out-of-control signals are produced until the new physical state of the process is represented in the reference sample. As it is known that there is a process change, these signals are not considered as false alarms.

Table 2 shows that many out-of-control signals are produced after depth 225 mm. These signals are caused by a change in the boundary conditions, see Section 2.1. Fig. 4 shows the means of the amplitudes of frequencies 234, 703 and 1183 Hz within segments of length 5 mm. Fig. 4 shows that the amplitudes of frequency 1183 Hz increase in the beginning of the drilling process. Transition to the second mode (703 Hz) occurs after depth 225 mm. In fact, the means
Table 2
Out-of-control signals of the different control charts

<table>
<thead>
<tr>
<th>Hole depth (mm)</th>
<th>Observation number</th>
<th>rMEWMA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\lambda = 0.1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\lambda = 0.1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$MD$</td>
</tr>
<tr>
<td>$\leq 32$</td>
<td>$\leq 106$</td>
<td>0</td>
</tr>
<tr>
<td>32–50</td>
<td>107–166</td>
<td>52</td>
</tr>
<tr>
<td>50–75</td>
<td>167–249</td>
<td>7</td>
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<tr>
<td>75–100</td>
<td>250–332</td>
<td>0</td>
</tr>
<tr>
<td>100–125</td>
<td>333–416</td>
<td>0</td>
</tr>
<tr>
<td>125–150</td>
<td>417–499</td>
<td>0</td>
</tr>
<tr>
<td>150–175</td>
<td>500–582</td>
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<tr>
<td>175–200</td>
<td>583–665</td>
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<td>200–225</td>
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<td>225–250</td>
<td>750–832</td>
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<td>250–275</td>
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<td>275–300</td>
<td>916–998</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>77</td>
</tr>
</tbody>
</table>

Fig. 4. Means of the amplitudes of frequencies 234, 703 and 1183 Hz within segments of length 5 mm.

of the amplitudes of frequency 703 Hz increase and dominate the process after depth 225 mm. The results show that all control charts detect this change quickly. Moreover, Fig. 4 shows that the increase in the amplitude of frequency 703 Hz is faster after depth 253 mm. This means that the transition from stable drilling to chatter starts at that depth. All control charts detect the start of the transition and many out-of-control signals are produced until depth 300 mm. In conclusion, in this experiment chatter vibration may be avoided if corrective actions are taken after these signals.

All control charts did not signal at $100 \leq depth \leq 125$ mm. It is known that depth 110 mm is approximately the position where the tool enters the bore hole completely. Theis (2004) noted that this might lead to changes in the dynamics of the process because the boring bar is slightly thinner than the tool and therefore the pressures in the hole may change. Fig. 4 shows that there is a decrease in the amplitudes of frequency 1183 Hz after depth 110 mm. This explains that all control charts fail to detect this change.

5. Discussion

5.1. The masking problem

In this experiment, the residuals are calculated using $k = 5$ in Eq. (5). In fact, the rMEWMA control charts used to monitor the residuals suffer from a masking problem. Table 3 shows the out-of-control signals of the rMEWMA control charts at $250 \leq depth \leq 275$ mm. The smoothing parameter is $\lambda = 0.3$ and values of $k = 1$ and 5 are used in Eq. (5).
Table 3
Out-of-control signals of the two rMEWMA control charts using $k = 1$ and $k = 5$ in Eq. (5) ($250 \leq \text{depth} \leq 275$ mm)

<table>
<thead>
<tr>
<th>Observation number</th>
<th>Hole depth</th>
<th>rMEWMA $k = 5$</th>
<th>rMEWMA $k = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$MD$</td>
<td>$SD$</td>
</tr>
<tr>
<td>842</td>
<td>252.91</td>
<td></td>
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</tr>
<tr>
<td>843</td>
<td>253.21</td>
<td></td>
<td></td>
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<tr>
<td>844</td>
<td>253.51</td>
<td>$\times$</td>
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<td>$\ldots$</td>
<td>$\times$</td>
<td></td>
</tr>
<tr>
<td>847</td>
<td>254.42</td>
<td>$\times$</td>
<td></td>
</tr>
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<td>848</td>
<td>254.72</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\times$</td>
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<tr>
<td>885</td>
<td>265.83</td>
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<tr>
<td>893</td>
<td>268.23</td>
<td>$\times$</td>
<td></td>
</tr>
<tr>
<td>894</td>
<td>268.53</td>
<td>$\times$</td>
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<td>$\ldots$</td>
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<tr>
<td>903</td>
<td>271.24</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
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<td>$\ldots$</td>
<td>$\times$</td>
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</tr>
<tr>
<td>915</td>
<td>274.84</td>
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Fig. 5. Plot of $E_t = \{e_{t-100+1}, \ldots, e_t\}$ (a) $t = 842$ and (b) $t = 843$.

The rMEWMA control charts based on $MD$ and $SD$ with $k = 5$ detect the start of the transition from stable drilling to chatter after 2 and 6 observations, respectively. However, the rMEWMA control charts with $k = 1$ did not detect the change. The first out-of-control signals are produced after 61 observations (depth 271 mm).

The reason is that the change is quickly transferred to the adaptive estimated parameters. In fact, one limitation of the use of adaptive estimates to calculate the residuals is the “masking” or parameter adaptation problem. If an early process change is not quickly detected, then the parameter estimates may be adversely affected by the change, thus masking the shift from future detection. For example, using $k = 1$, Fig. 5 shows that observation $e_{842}$ is outside the data cloud. Its simplicial depth and rank are equal to 0.03 and 3.5, respectively, see Table 4. This causes the rMEWMA control statistic to wander to the direction of an out-of-control. However, the change is quickly transferred to $\phi_{1843}$ and $\beta_{843}$. Therefore, observation $e_{843}$ is deep in the data cloud or reference sample, see Fig. 5. Its rank is equal to 91, which cause the rMEWMA control statistic to wander in a direction opposite of an out-of-control, see Fig. 6.
Table 4
Illustration of the masking problem

<table>
<thead>
<tr>
<th>Observation number (t)</th>
<th>Hole (mm)</th>
<th>1183 Hz</th>
<th>703 Hz</th>
<th>SD</th>
<th>$Q_t^+$</th>
<th>$Q_t^-$</th>
<th>EWMA $T_t$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta_t$</td>
<td>$\phi_{1,t}$</td>
<td>$\beta_t$</td>
<td>$\phi_{1,t}$</td>
<td></td>
<td></td>
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<tr>
<td>836</td>
<td>251.11</td>
<td>0.179</td>
<td>0.119</td>
<td>0.447</td>
<td>0.026</td>
<td>0.104</td>
<td>53</td>
</tr>
<tr>
<td>837</td>
<td>251.41</td>
<td>0.181</td>
<td>0.105</td>
<td>0.448</td>
<td>0.025</td>
<td>0.098</td>
<td>50</td>
</tr>
<tr>
<td>838</td>
<td>251.71</td>
<td>0.183</td>
<td>0.096</td>
<td>0.449</td>
<td>0.023</td>
<td>0.100</td>
<td>50</td>
</tr>
<tr>
<td>839</td>
<td>252.01</td>
<td>0.182</td>
<td>0.106</td>
<td>0.451</td>
<td>0.021</td>
<td>0.214</td>
<td>88</td>
</tr>
<tr>
<td>840</td>
<td>252.31</td>
<td>0.180</td>
<td>0.114</td>
<td>0.451</td>
<td>0.037</td>
<td>0.030</td>
<td>4.5</td>
</tr>
<tr>
<td>841</td>
<td>252.61</td>
<td>0.181</td>
<td>0.113</td>
<td>0.434</td>
<td>0.088</td>
<td>0.065</td>
<td>36</td>
</tr>
<tr>
<td>842</td>
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<td>0.180</td>
<td>0.106</td>
<td>0.400</td>
<td>0.203</td>
<td>0.030</td>
<td>3.5</td>
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<tr>
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<td>252.21</td>
<td>0.181</td>
<td>0.116</td>
<td>0.382</td>
<td>0.238</td>
<td>0.232</td>
<td>91</td>
</tr>
<tr>
<td>844</td>
<td>253.51</td>
<td>0.181</td>
<td>0.120</td>
<td>0.370</td>
<td>0.288</td>
<td>0.043</td>
<td>23</td>
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</tbody>
</table>

Fig. 6. Plot of the rMEWMA with $\lambda = 0.3$ (836 \(\leq t \leq 845\)).

5.2. Multiple frequency modelling

In Section 2.2, the van der Pol model is used to describe the transition from stable drilling to chatter vibration in one frequency. Therefore, the amplitudes of the relevant frequencies are modelled separately. As noted by Weinert et al. (2004), the van der Pol model accounts for a subset of experimental results, especially when only one frequency dominates chatter vibration. Naturally, the model fails to capture all aspects of the process when this assumption becomes invalid. It is always the case when chatter vibration at the beginning of the drilling process is observed. Indeed, two or three frequencies dominate the process and they affect each other either by excitatory or inhibitory interaction. Therefore a more complicated model that include the interactions between the relevant frequencies is needed. In this case a multivariate time series model can be used instead to calculate the residuals. For process monitoring, the proposed rMEWMA control chart can be easily used to monitor such residuals or the multivariate time series modelling can be combined with control charts to monitor the process. For example, Jarrett and Pan (2007) proposed vector autoregressive control charts for multivariate autocorrelated processes.

5.3. Simplicial depth computation

For the simplicial depth computation in $\mathbb{R}^2$, Rousseeuw and Ruts (1996) proposed an efficient algorithm to compute the simplicial depth of a point or position using Liu’s definition. It has a time complexity $O(n \log n)$, where $n$ is the
size of the reference sample. **Burr et al. (2004)** proposed certain modifications to this algorithm in order to compute the revised simplicial depth. When the point or position is in \( \mathbb{R}^3 \), **Cheng and Ouyang (2001)** proposed an \( O(n^3) \) time algorithm. They generalized it to \( \mathbb{R}^4 \) with a time complexity \( O(n^4) \). These two algorithms are based on Liu’s definition. **Burr et al. (2004)** showed that these algorithms remain valid under the revised definition with some adjustments. For \( \mathbb{R}^d \) \((d \geq 5)\), there are no known algorithms faster than the straightforward method. That is to generate all the simplices and count the number of containments. It is extremely computationally intensive. It has a time complexity \( O(n^{d+1}) \) which constitutes an obstacle for the use of simplicial depth in practice.

### 6. Conclusion and future work

In this experiment, the \( r \)-MEWMA control charts with \( \lambda = 0.3 \) are the best, and should be chosen among the different control charts considered in this work. Indeed, all physical changes of the drilling process are detected, except when the tool enters completely the hole. Also, the start of the transition from stable drilling to chatter vibration is detected quickly. In practice, a procedure to choose the smoothing parameter \( \lambda \) is required.

For the process adjustment, once the \( r \)-MEWMA control chart has produced a signal, procedures to estimate the shift magnitude, to identify the point in time at which the shift has occurred and to interpret the out-of-control signal are needed. Firstly, one limitation of the \( r \)-MEWMA control charts is that they do not give an information about the shift magnitude. Secondly, the identification of the time point at which the shift has occurred may help the process engineers to adjust the process. In this example, the start of the transition from stable drilling to chatter is detected after two observations. Finally, the out-of-control interpretation is basic for the adjustment of the process. In fact, when the control chart indicates an out-of-control condition, it is important to determine which frequency, or combination of frequencies, of the multivariate process caused the process to go out-of-control. For example, **Berismis et al. (2005)** gave an overview of the proposed procedures for interpreting an out-of-control signal in a multivariate control chart. In practice, the identification of the type of chatter (i.e. chatter at the beginning of the drilling process, low-high frequency chatter) will usually make it easier for engineers to adjust the process. Note that chatter suppression may be achieved by decreasing the cutting speed or varying the feed rate.

Future research should focus on the selection of the adequate data depth measure, which is an important issue for the success of the proposed monitoring strategy.

### Acknowledgments

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### References


