Chatter Stability of Plunge Milling

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Abstract

Plunge milling operations are used to remove excess material in boring cylinders, roughing pockets, dies and mold cavities. This paper presents a frequency domain, chatter stability prediction theory for plunge milling. The regenerative chip thickness is modeled as a function of lateral, axial and torsional vibrations. The stability of the plunge milling is formulated as a fourth order eigenvalue problem by relating the regenerative chip thickness, cutting forces and torque, and the structural modes of the cutter. The stability lobes are predicted analytically from the eigenvalue solution. The stability lobes are experimentally proven by conducting over one hundred plunge milling tests.

Keywords:
Milling, Chatter, Stability

1 INTRODUCTION

The plunge milling operations are conducted by feeding the cutter into solid blocks like a drill, or to enlarge a hole like in boring with indexed boring heads, or by removing parts of the material intermittently in axial direction. Since the spindle is most rigid in the axial direction where the chip thickness is measured and the major cutting force acts, plunge milling operations became quite popular in roughing the cavities of dies, molds and aerospace parts made from hardened steel or thermally resistant alloys. The concern is not to chip tools and to avoid overloading of the tool holder and spindle trust bearing load limits, which occur easily if chatter appears during cutting. Chatter stability of machine tools has been studied since 1950s with the pioneering works of Tobias [1] and Tlusty [2]. They both assumed that the cutting point is stationary and the process dynamics are time invariant, which is called one dimensional chatter by Altintas [3]. An extensive review of chatter stability in metal cutting and grinding were presented by Altintas and Weck in a CIRP key note paper [3], and will not be repeated here. The chatter stability in milling is predicted either in time or frequency domains. Time domain simulation of the process involves numerical modeling of the process physics with nonlinearities by replicating series of cutting tests [4,5]. Alternatively, the linear stability equation, which contains differential equations with delay terms, is analytically solved in time domain as presented in [6,7]. Altintas and Budak [8, 9] solved the chatter stability of peripheral milling in frequency domain, which leads to direct relationship between the depth of cut and speed. They considered flexibilities and regeneration in two lateral directions (x,y), hence the stability was two dimensional. In this paper, the formulation is extended to fourth order by considering the influence of vibrations in two lateral, axial and torsional directions, which are applicable to plunge milling, multi-inserts boring and indexed drilling operations.

2 CHATTER STABILITY OF PLUNGE MILLING

The geometry of a plunge milling cutter is shown in Figure 1. The tool cuts the metal at the bottom cutting edges of the inserts as it plunges into the metal vertically. The inserts have an offset distance of I from the cutter center.

Figure 1: Geometry and coordinates of a sample plunge mill: Shank Diameter \(D_1=20\, mm\), Cutting Tool Diameter \(D_2=25\, mm\), I=5.5 mm, Axial rake angle \(\alpha_f=10\, deg\), \(\psi_r=10\, deg\), Axial relief angle \(Cl_f=5\, deg\). (Sandvik Part Number: R210-025A20-09M)
The tangential cutting force \((F_t)\) acts in the direction of cutting speed. The feed force \((F_f)\) is in the direction of feed or \(z\) axis, and the passive force \((F_p)\) acts along the cutting edge. The angular position \(\phi(t)\) of the cutting edge \((j)\) at time \(t\) is evaluated using \(y\) axis as a reference (Figure 1):

\[
\phi(t) = \Omega t + (j - 1)\phi_p
\]

where \(\Omega\) is the angular speed of the spindle in rad/s, \(\phi_p = 2\pi/N\) is the pitch angle, and \(N\) is the number of teeth.

The dynamic chip thickness removed by insert \(j\) is expressed as a function of vibrations in three linear \((x,y,z)\) and torsional \((\theta)\) directions as:

\[
h_j = g_j |\Delta \phi(t)| \sin \phi(t) \tan \nu_r + |\Delta \theta(t)| \cos \phi(t) \tan \nu_r
\]

+ \(\Delta \phi(t) \cdot \Delta \theta(t) / (\phi_p)\c)

where \(g_j = 1\) when the insert is in cut, zero otherwise depending on the immersion conditions of the cutter.

\[
\Delta \phi = r(t) - r(t - T), \quad r = x, y, z, \theta \text{ terms are the regenerative vibrations in four directions, } T \text{ is the tooth period, and } c \text{ is the feed per tooth in plunge direction. Tangential \((F_t)\), radial \((F_r)\), feed forces \((F_f)\), and torque \((T_\theta)\) acting on the cutting edge are given by:}
\]

\[
F_j(t) = K_{nt} ab_j(t), \quad F_f(t) = K_{n} K_{nt} ab_j(t), \quad F_r(t) = K_{nt} K_{nt} ab_j(t), \quad T_\theta(t) = F_{r,j} \nu_k
\]

where torque arm is assumed to be measured from the center of the cutter insert \(r_i = (D_2 + 2L)/4\).

The total instantaneous cutting forces and torque in Cartesian coordinates can be evaluated by summing the contributions of all inserts in cut:

\[
\begin{bmatrix}
F_x(t) \\
F_y(t) \\
F_z(t) \\
T_\theta(t)
\end{bmatrix}
= \sum_{j=1}^{N} \begin{bmatrix}
-\cos \phi_j - \sin \phi_j & 0 & 0 & F_{y,j} \\
\sin \phi_j & -\cos \phi_j & 0 & F_{z,j} \\
0 & 0 & 1 & 0 & F_{z,j} \\
0 & 0 & 0 & 1 & T_{\theta,j}
\end{bmatrix}
\]

By substituting dynamic chip thickness Equation 2 and rotating insert forces Equation 3 into Equation 4, the equation of motion with regenerative chip removal mechanism is expressed as:

\[
\begin{bmatrix}
F_x(t) \\
F_y(t) \\
F_z(t) \\
T_\theta(t)
\end{bmatrix}
= aK_{nt} \sum_{j=1}^{N} \begin{bmatrix}
-\cos \phi_j - K_{nt} \sin \phi_j \\
\sin \phi_j & -K_{nt} \cos \phi_j \\
0 & 0 & 1 & 0 & F_{z,j} \\
0 & 0 & 0 & 1 & T_{\theta,j}
\end{bmatrix}
\] \(\Delta \phi_j\)

\[
[\sin \phi_j \tan \nu_r \cos \phi_j \tan \nu_r 1 - 1/2 \pi c N]
\]

or in a matrix notation form:

\[
\{F(t)\} = aK_{nt} [\Delta(t)] \{\Delta(t)\}
\]

where \([\Delta(t)]\) is periodic, time varying directional factors due to cutter rotation \(\phi(t) = \Omega t + (j - 1)\phi_p\). Terms in \([\Delta(t)]\) are dependent on the tool geometry and engagement of the plunge mill with the part. The time variation of the directional factors requires either multi-frequency solution as proposed by Budak et al. [9], or time domain solution of delayed differential equations as presented in [7,8,10]. However, when the periodic directional factors do not contain short impulse type wave forms, Altintas et al. [3] showed that their mean values lead to as accurate solution as the case when the time varying factors are fully considered. Since \([A(t)]\) is periodic at tooth passing interval \(T\) or pitch angle interval \(\phi_p\), the mean values can be evaluated as:

\[
[A_0] = \frac{1}{T} \int_{0}^{T} \{A(t)\} dt = \frac{1}{\phi_p} \int_{\phi_p}^{2\pi} \{A(\phi)\} d\phi = N/2\pi [a]
\]

The vibrations in each direction are related to cutting forces and torque through the Frequency Response Function (FRF) of the machine tool identified at the cutter.

\[
\begin{bmatrix}
\chi \\
\nu \\
\theta
\end{bmatrix}
= \begin{bmatrix}
\Phi_{xx} & 0 & 0 & 0 & F_x \\
0 & \Phi_{yy} & 0 & 0 & F_y \\
0 & 0 & \Phi_{zz} & \Phi_{z\theta} & F_z \\
\Phi_{x\theta} & 0 & \Phi_{y\theta} & 0 & T_\theta
\end{bmatrix}
\]

or

\[
\{r(t)\} = \{\Phi(\omega)\} \{F(t)\}, \quad \{r(t - T)\} = \{\Phi(\omega)\} e^{-i\omega T} \{F(t)\}
\]

where \(\{r(t)\} = [\chi(t), \nu(t), \theta(t)]^T\) represents the translational and torsional vibrations produced by cutting forces \([F(t) = \{F_x, F_y, F_z, T_\theta\}]\). \(\{r(t - T)\}\) represents the vibrations generated during the previous tooth period \((T)\). The FRF matrix \([\Phi(\omega)\]\ contains direct \((\Phi_{xx}, \Phi_{yy}, \Phi_{zz}, \Phi_{x\theta})\) and cross \((\Phi_{x\theta}, \Phi_{y\theta})\) receptances of the plunge mill at its tip, and are expressed by their modal parameters in the following form:

\[
\Phi_{ HH}(\omega) = \sum_{k=1}^{m} \frac{\alpha_{ab}^2}{\omega_k^2 - \omega^2 + 2\zeta_k \omega_k \omega}
\]

where \(m\) is the total number of modes in direction \(r\) and \(\alpha_{ab}\), \(\zeta_k\) are the natural frequency, modal stiffness, and damping ratio of mode \(k\), respectively. The axial and torsional vibrations are coupled in this particular cutter due to large flute cavities for chip evacuation, and they are represented by cross transfer functions \((\Phi_{x\theta}, \Phi_{y\theta})\). The FRFs can either be identified through impact modal tests applied on the cutter tip while attached to existing machines, or predicted through Finite Element analysis applied on the digital model of the conceptual machine tools.

When the plunge milling process is critically stable, the regenerative vibrations occur at vibration frequency \(\alpha_k\) with constant amplitude.

\[
\{\Delta(t)\} = \{r(t)\} - \{r(t - T)\} = (1 - e^{-i\alpha_k T}) \{\Phi(\alpha_k)\} \{F(t)\}
\]

By substituting the regenerative vibration vector Equation 11 into dynamic plunge milling Equation 6 with average directional factors (Equation 7), the following eigenvalue equation is established for plunge milling stability:

\[
\{F_e\} e^{-i\omega T} = aK_{nt} (1 - e^{-i\omega T}) [\{\alpha \Phi(\omega)\}] \{F_e\} e^{i\omega T}
\]
By defining the oriented frequency response function matrix (Φ) and the eigenvalue (Λ) of the characteristic equation as:

\[ Φ_0 = [x][Φ(ω_0)]_1, \quad Λ = -\frac{N}{2\pi}aK_c(1-e^{-iωT}) \]  

(13)

The resulting characteristic equation (Equation 12) becomes,

\[ \det[I + ΛΦ(ω_0)] = 0 \]  

(14)

Since the oriented transfer function matrix (Φ_0) is fourth order, the eigenvalues, which lead to critically stable cutting conditions, are found from the roots of the following polynomial:

\[ a_0Λ^4 + a_1Λ^3 + a_2Λ^2 + a_3Λ + 1 = 0 \]  

(15)

Each eigenvalue has a real and imaginary part:

\[ Λ = Λ_r + iΛ_i = -\frac{N}{2π}aK_c(1 - e^{-iωT}) \]  

(16)

The details of the identification of critical depth of cut (a_{lim}) and spindle speed (n) to construct the lobes can be found in [8]. The critical depth of cut is found as:

\[ a_{lim} = \frac{πκR}{N/K_c}(1 + κ^2) \quad ← κ = Λ_r - \frac{πω_0T}{1 - \cosω_0T} \]  

(17)

The spindle speed n (rev/min) is found from the identified delay or the tooth passing period T(second)

\[ T = \frac{1}{ω_0}(ε + 2πκ) \quad ← ε = π - 2\tan^{-1}κ, \quad \to n = \frac{60}{NT} \]  

(18)

The eigenvalues (Λ) are solved by scanning possible chatter frequencies from the FRFs to generate pairs of critical depth of cut and corresponding speeds for each lobe k=1,2,3,..., as explained in [8].

### 3 SIMULATION AND EXPERIMENTAL RESULTS

The FRFs of the plunge mill attached to Mori Seiki SH403 machining center with HSK63E interface are identified through impact modal tests. The most flexible modes, which contribute to the regeneration mechanism, are considered as shown in Table 1. The torsional mode is measured by attaching a miniature accelerometer on one tooth while exerting impact blows on the opposite tooth with a miniature hammer. The torsional-axial mode is also measured by the same instruments. The impact is applied on the tool radially while accelerometer mounted on the axial direction of the opposing tooth.

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Natural Frequency (Hz)</th>
<th>Damping Ratio (ζ)</th>
<th>Dynamic Stiffness (2ζ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_1</td>
<td>508</td>
<td>0.024</td>
<td>5.68 N/μm</td>
</tr>
<tr>
<td>x_2</td>
<td>3040</td>
<td>0.024</td>
<td>1.99 N/μm</td>
</tr>
<tr>
<td>x_3</td>
<td>4035</td>
<td>0.018</td>
<td>2.42 N/μm</td>
</tr>
<tr>
<td>y_1</td>
<td>515</td>
<td>0.051</td>
<td>4.52 N/μm</td>
</tr>
<tr>
<td>y_2</td>
<td>3100</td>
<td>0.030</td>
<td>4.90 N/μm</td>
</tr>
<tr>
<td>y_3</td>
<td>3961</td>
<td>0.018</td>
<td>1.87 N/μm</td>
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<tr>
<td>z_1</td>
<td>321</td>
<td>0.048</td>
<td>23.30 N/μm</td>
</tr>
<tr>
<td>z_2</td>
<td>405</td>
<td>0.038</td>
<td>59.80 N/μm</td>
</tr>
<tr>
<td>z_θ</td>
<td>12090</td>
<td>0.0015</td>
<td>0.0649 Nm/μm</td>
</tr>
<tr>
<td>θ_θ</td>
<td>12000</td>
<td>0.0024</td>
<td>53.80 Nm/μm</td>
</tr>
</tbody>
</table>

Table 1: Modal parameters of the plunge mill attached to Mori Seiki SH403 with HSK 63 interface.
The spectrum of the sound shows harmonics which are spread on either side of the chatter frequency at tooth passing frequency (533.3Hz) intervals [9]. The measured cutting forces for the chatter free cutting conditions at 17142 rev/min with 5.75mm radial depth of cut are shown in Figure 4. The process is stable, the cutting forces contain only quasi-static, periodic behavior at tooth passing intervals at this spindle speed of 17142 rev/min.

4 CONCLUSION
Dynamics of the plunge milling need to be represented by considering the influence of vibrations in two lateral (x,y), one axial (z) and torsional directions in order to model cutters with general geometry and boring operations. In this paper, the critical depth of cut and spindle speeds are identified by solving the stability of a fourth order, coupled, delayed differential equations in frequency domain. When the cutter geometry has large chip evacuation cavities between the teeth, the tool exhibits strong torsional vibrations resembling twist drills. The torsional vibrations are transmitted as axial deflections, which directly change the regenerative chip thickness, and becomes a dominant source of chatter. Flexibilities in other directions may be important as well, if their vibrations are transmitted to the chip thickness direction by the cutter geometry more.

By changing the engagement conditions between the part and workpiece, the proposed model can be used to predict the chatter stability of boring cylinders with indexed heads, plunging into solid workpiece, or removing part of the material by partially engaging the cutter periphery with the workpiece.

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6 REFERENCES